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Review
on the PhD thesis
«Quantum calculus and its application to fractional differential equations»
by NarimanTokmagambetov

This PhD thesis is devoted to quantum calculus and related applications in fractional differential equations.

Nowadays there is a considerable interest in these topics. One important reason for this fact is that this theory is not only very important in itself but for various applications both in mathematics, technical and natural sciences, e.g. in computer science, quantum mechanics and quantum physics.

Thus, there has been a significant development in the field of simple differential and partial differential equations, and a large number of papers and several books have been published on this topic in various fields. However, the theory of difference equations with constant and variable coefficients is still in its early development, and many aspects of this theory still need to be investigated. Therefore, the use of quantum calculus, including q-calculus, is relevant in finding solutions to fractional differential equations.

Application of the q-calculus of quantum calculus in differential equations with fractional derivatives, especially by finding their solutions and proving their existence and uniqueness, is the aim of this PhD thesis.

The PhD thesis consists of three Chapters.

In Chapter 1 all the formulas, definitions and lemmas that are necessary to prove the theorems in Chapters 2 and 3 are obtained.

In the first part of the Chapter 2, a nonlinear problem of the Cauchy type with a q-fractional Riemann-Liouville derivative is considered. First, an equivalence theorem is proved, which is of independent interest. Next, a theorem on the existence and uniqueness of a unique global solution of the Cauchy problem in the space $L^1_{q,a+}[a,b]$ for $\alpha > 0$ is proved. The q-analogue of the composite fractional operator or Hilfer derivative operator is also defined. Definitions of the fractional Hilfer q-derivative and the generalized fractional q-derivative are given. To prove the main results, two new lemmas of independent interests are proved. Some new equivalence theorems for the q-fractional problem of Cauchy type and the q-integration Volterra equation are proved.

Chapter 3 is devoted to derive both explicite and numerical solutions of some linear-fractional q-difference equations and Cauchy-type problems related to the fractional Riemann-Liouville q-derivative in q-calculus. These new methods,

which are based on Voltaire's q-integral equations, compositional relations, and operational calculations, have been derived in order to obtain specific solutions to linear q-difference equations. Some of the results have been presented that include fractional q-difference equations of order $\alpha > 0$ and given real numbers in the q-calculus, as well as numerical processing of fractional q-difference equations. In addition, some real solutions of linear-fractional q-differential equations with the Caputo q-fraction of order $\alpha > 0$ were created and investigated. In the final Section of the Chapter, concrete solutions are obtained for a new modification of the Schrödinger equation related to the q-Bessel operator. A theorem on the existence of this solution in the q-calculus in a Sobolev type space is proved. Thus, in particular, in each Section there are given examples illustrating the new main results.

I judge that this study is fundamental in many respects and has made a great contribution to the development of q-calculus in fractional differential equations. In particular, some main results of this PhD thesis are already published in the high-ranked journal "Filomat" (Scopus, Web of Science), and the candidate has presented them at some international mathematical conferences and seminars.

Summing up, I consider that concerning the scientific significance and novelty of the obtained results, it is no hesitation that the PhD thesis of **Nariman Tokmagambetov** on the theme "Quantum calculus and its application to fractional differential equations" satisfies all the requirements for a PhD thesis of very good international standard. Hence, I strongly recommend that the candidate will be awarded the title Doctor of Philosophy (PhD) in the specialty "6D060100-Mathematics".

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